Simulation of Flow in Vent System

Vent Software Tutorial

2008
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Introducing Vent

VENT is powerful software for simulating a flow through a hydraulic line. It is mainly intended for designing so-called “vent systems” or “vent lines”. Vent system is a hydraulic line mounted to chemical equipment (reactors, tanks, columns, etc.) for prevention of tank bursting in case of a runaway reaction that may occur because of insufficiency of external cooling.

The VENT software uses some DIERS\(^1\) recommendations for piping design as well as some original algorithms. The flow is assumed to be quasi-stationary and one-dimensional. It can consist of gas and/or liquid phase. Currently, VENT supports mixtures of up to 9 species. The vent line is single-duct and complex. It can include up to 256 elements. VENT allows calculation of a flow under a wide range of initial conditions.

VENT is a component of the software system for evaluation of thermal safety of chemical processes (TSS) developed by CISP, but is also available either as a stand-alone application or integrated in the BST\(^2\) or other software.

Basic Features

VENT simulates a multi-component gas-liquid flow through a complex single-duct hydraulic line. The simulation is performed by numerical solution of a system of ordinary differential equations. Since the flow is assumed to be quasi-stationary, VENT does not calculate any time dependencies. Since the flow is assumed to be one-dimensional, the flow parameters are calculated only along the line length.

The initial data for calculations are the following:

- The structure of the vent line. The structure is defined by the number of elements, their types (subtypes) and order. The following elements can be simulated: straight tube, conic tube, short nozzle, nozzle, expansion/contraction, elbow, nonreclosing device, conventional valve, balanced valve, pilot valve, other valve, low pressure device, orifice, tee, fitting.
- Geometric parameters for each element of the line.
- Hydraulic parameters for each element of the line (either roughness or discharge coefficients for liquid and gas phases (or Hooper coefficients)).
- Set pressure (for all types of nonreclosing devices, all types of valves and other low pressure devices), full lift overpressure and reclosing pressure (for all types of valves).
- Additional miscellaneous info for each element (optional).
- Flow model type and parameters. The following models are supported: slip equilibrium model (for long tubes), homogeneous equilibrium and homogeneous frozen models (for short nozzles). The parameters are methods for calculation of fanning factor and slip ratio (only for the slip equilibrium model).
- Calculation accuracy.
- Flow composition: names of individual species composing the flow (selected by means of the MIXTURE\(^3\) software from its database), their mass fractions in liquid and gas phases, void fraction or quality of the flow, and physical properties of the flow mixture (automatically calculated by the MIXTURE software).
- Boundary conditions: inlet pressure, inlet temperature, back pressure.
- Project comment (optional).

\(^1\) Design Institute for Emergency Relief Systems

\(^2\) Design Institute for Emergency Relief Systems

\(^3\) MIXTURE is a software for evaluation of physical properties of liquid and gas ideal and non-ideal mixtures in a wide range of temperatures and pressures. It is a component of the software system for evaluation of thermal safety of chemical processes (TSS) developed by CISP. MIXTURE is always delivered as a part of the VENT package, but can be also used autonomously.
The following data can be obtained as a result of simulation:

- Dependencies of the flow rate on the inlet pressure, back pressure, inlet temperature and mass fraction of gas phase (flow quality).
- Distributions of pressure in the pipe, flow quality and void fraction, temperatures of liquid and gas phases, specific discharge, flow rates of the liquid and gas phases along the line length at various inlet pressures, inlet temperatures and back pressures.
- Values of thrust for each element of the vent line at various inlet pressures, inlet temperatures and back pressures.

**Scope of this Chapter**

This chapter presents theoretical backgrounds of VENT, including the models it uses in order to simulate a flow and formulas for calculating fanning factor and slip ratio. If you encounter any problems you fail to solve with the help of the VENT documentation or have any suggestions on improving the software and its backgrounds, please do not hesitate to contact us:

The current version of VENT supports 3 models. Namely, they are slip equilibrium model (SEMB) for long tubes, homogeneous equilibrium models (HEM) for real and ideal gases, and frozen equilibrium model for a short nozzle.

Note that Slip Equilibrium model is not really a model but the whole class of ones. In fact, there are 24 various models with similar algorithms. They differ in a way to calculate the fanning factor and slip ratio between the phases. See the details in corresponding section.

We recommend applying HEM models if only the rough estimate is needed or if there is a system with a single nozzle and some tubes that can be neglected.

---

4 There will be more models in the next version. For sure, some simplified algorithms for long tubes, such as the omega method, will be included.

5 The homogeneous equilibrium model for long tubes is considered as a specific case of SEMB, i.e. SEMB with unit slip ratio or with “homogeneous slip ratio”.
**Homogeneous equilibrium model for ideal gas and homogeneous frozen model**

The standard homogeneous model for a short nozzle from paper [1] cannot be applied to the cases of frozen liquid or presence of insoluble gases. However, applying assumptions for the previous model and neglecting friction for the short channels, we get the universal model in the ideal gas approximation that can be applied to any case. As we have mentioned, the problem statement is similar to that for the standard model for boiling liquid without insoluble gases, which vapor phase is considered to be ideal gas. The frozen homogeneous model differs from this one in another assumption that the gas quality in the flow is constant and equal to gas quality at the system inlet. So, there are no terms relevant to vapor emission at boiling in corresponding equations.

The impulse equations are as follows:

For frozen liquid at \( P > P_s(T_0) \)

\[
(g \cdot v)^2 = I_1 \cdot X_{g0} + (1 - X_{g0}) \cdot I_2
\]

\[
I_1 = v_{g0} \left| _{p = p_0} \right. \cdot \frac{2 \cdot k \cdot p_0}{k - 1} \left( 1 - \eta^{1 - \frac{1}{k}} \right)
\]

\[
\eta = \frac{p}{p_0};
\]

\[
I_2 = 2 \cdot \bar{v}_f \cdot (p_0 - p)
\]

For boiling liquid at \( P \leq P_s(T_0) \)

\[
(g \cdot v)^2 = I_1 \cdot X_{g0} + (1 - X_{g0}) \cdot I_2
\]

\[
I_1 = v_{g0} \left| _{p = p_0} \right. \cdot \frac{2 \cdot k \cdot p_0}{k - 1} \left( 1 - \eta^{1 - \frac{1}{k}} \right)
\]

\[
\eta = \frac{p}{p_0};
\]

\[
I_2 = 2 \cdot \bar{v}_f \cdot (p_0 - p) + i_{\text{froz}} \cdot 2 \cdot \bar{c}_p \left( T_s - T_0 + T_0 \cdot \ln \left( \frac{T_0}{T_s} \right) \right)
\]

\[
T_s = \frac{T_0}{1 - \frac{p s (T_0)}{T_0} \frac{d T_s}{d p} \left| _{0}^{\eta_s} \right. \cdot \ln \eta_s}; \quad \eta_s = \frac{p}{P_s(T_0)}
\]

\( i_{\text{froz}} = 0 \) for frozen model and \( i_{\text{froz}} = 1 \) for equilibrium model

Let us list the notation:

Equations (1) and (2) are solved together with the critical condition for equilibrium homogeneous critical flow for ideal gas:

For frozen liquid:

\[
g_{cr}^2 \approx \frac{1}{v_{g0} \cdot X_{g0} + (1 - X_{g0}) \cdot \bar{v}_f \cdot \beta_{p_f}}
\]

For boiling liquid at \( P \leq P_s(T_0) \)
That is the HEM or HFM model for ideal gas

Here $X_g = (1 - X_{g0}) \cdot X$; $C_{P_m} = C_{P_v} \cdot X + C_{P_f} \cdot (1 - X)$

Note, that $X_{g0}$ depends on the mass discharge through the nozzle $W$, where $W = g \cdot A_{lim}$, where $A_{lim}$ is the least flow area of the nozzle

**Slip equilibrium models (SEMB)**

Inhomogeneous equilibrium quasi-bubble models (SEMB) for estimating a two-phase flow through long pipes of various forms assume the following:

- slip case with various ratios of vapor-to-liquid mean velocities;
- gas and vapor are of the same temperature (immediate mixing);
- temperature of flashing liquid is in equilibrium with partial pressure of vapors in gas phase

Let us consider the phases motion with slipping.

To obtain the necessary equations, let the flow path length in the pipe be considered as an independent variable. For any other variable $y$: $y' = \frac{dy}{dl}$

The stagnation parameters in the tank are used as initial ones at the pipe inlet. If the pressure drop is not high enough to attain maximum (critical) flow, the conditions at the pipe end are:

$$P_2 = P_b.$$ 

Here $P_b$ denotes the backpressure value; otherwise the pressure at the pipe outlet is determined by maximum (critical) flow conditions.

$$\left( (W_i \cdot u_i) + (W_y \cdot u_v) + (W_g \cdot u_g) \right)' dx + P_{x + dx} \cdot A_x + P_x \cdot A_x \frac{P_{x + dx} + P_{x + dx}}{2} \frac{dA}{dx} dx - F_{fric} \cdot dx - F_g \cdot dx = 0$$

$$u_g = \frac{W \cdot \bar{v}_s}{A}; \quad \alpha = \frac{1 - X_1}{1 - X_1 + \frac{\rho_{vg} \cdot X_1}{\rho_1} \cdot s}; \quad \bar{v}_s = \frac{X_{gv}}{\rho_{gv}} + \frac{X_1}{\rho_1} \cdot s;$$

The momentum balance equation for a section of a pipe may be as follows:
A new approach has been developed.

Since \( \bar{X}_{gs} \cdot \bar{v}_S \) under barotropic assumption depends only on pressure, the last equation can be transformed to:

\[
\frac{dP}{dl} = -g^2 \frac{-\bar{v} \cdot A' + 2 \cdot f \cdot \Phi^2 \cdot W^2}{A \cdot D \cdot \rho \cdot A^2} \frac{g \cdot g_0 \cdot Z'}{1 - M^2}. 
\]

where \( M^2 = \frac{g^2}{g_{crit}^2}; \ g = \frac{W}{A}; \)

The critical flow is \( g_{crit}^2 \frac{dP}{dl} = -\frac{d\bar{v}}{dP}. \)

The momentum specific volume is

\[
\bar{v} \equiv \left( 1 - X_1 \left( 1 - \frac{1}{s} \right) \right) \left( v_{gs} \cdot X_{gs} + v_1 \cdot X_1 \cdot s \right); \quad (5)
\]

Holdup specific volume is

\[
\rho_h = \frac{\alpha}{v_{gs}} + \frac{1 - \alpha}{v_1}; \quad (6)
\]

\[
\alpha = \frac{1 - X_1}{1 - \frac{\rho_{gs}}{\rho_1} \cdot X_1 \cdot s} \quad (7)
\]

**The energy balance equations**

Let us assume that variation of kinetic energy is much less than that of enthalpy. For subcritical and critical

\(^6\) For bottom elevation, the liquid head pressure should be accounted. \((\rho_h \cdot g_0 \cdot Z)\)
The energy equation for the liquid can be represented in the following form:

\[
(W_{gl} \cdot H_{gl}) + (W_l \cdot H_1) \equiv 0; \Rightarrow \\
H_{gl} \cdot X_{gl} + H_1 \cdot X_1 + \Delta H_{gl} \cdot G_s \equiv 0
\]

where

\[
\Delta H_{gl} = H_{gl} (T_{gv}, c_{gv_i}, P) - H_1 (T_1, c_{l_1}, P) \\
= H_{gv} (T_{gv}, c_{gv_i}, P) - H_{gv} (T_s, c_{s_i}, P) + (H_{gv} (T_s, c_{s_i}, P) - H_1 (T_s, c_{l_1}, P)) \approx
\]

\[
C_p \left( \frac{T_{gv} + T_s}{2}, \bar{c}_{gv_i}, P \right) \left( T_{gv} - T_s \right) + \sum_{i=1}^{N_g} \frac{1}{c_{gv_i}} \left( c_{gv_i} - c_{s_i} \right) + L_s + \left( \frac{\partial H_{gv}}{\partial P} \right)_{T_s, c_{s_i}, P} - \left( \frac{\partial H_1}{\partial P} \right)_{T_s, c_{s_i}, P} \cdot (P - P_s).
\]

\[
H_1' = -\frac{G_s}{X_1} \left[ L_s + \left( \frac{\partial H_{gv}}{\partial P} \right)_{T_s, c_{s_i}, P} - \left( \frac{\partial H_1}{\partial P} \right)_{T_s, c_{s_i}, P} \right] (P - P_s) + C_{p_{gv}} \cdot \frac{\partial T_S}{\partial P} + \frac{\partial H_1}{\partial P} \cdot P' + \frac{L_S}{X_{gv}} \sum_{i=1}^{N_g} \frac{\partial H_{gv}}{\partial c_{gv_i}} \left( c_{gv_i} - c_{s_i} \right) + C_{p_{gv}} \cdot T'_{gv} \cdot X_{gv} + \frac{\partial H_1}{\partial P} \cdot P' \cdot X_{gv} \\
- \left( C_{p_{gv}} \left( \frac{T_{gv} + T_s}{2}, \bar{c}_{gv_i}, P \right) \left( T_{gv} - T_s \right) \right) \frac{C_{p_{gv}}}{L_S} \frac{\partial T_S}{\partial P} + \frac{\partial H_1}{\partial P} \cdot P' \equiv 0
\]

For a slip equilibrium model, temperature of flashing liquid is equal to the boiling point at the current partial pressure of vapors in the gas-vapor region, and composition of vapors generated at liquid gas surface is equilibrium at the same temperature and liquid composition. In this case we ignore the difference in composition in the core of gas-vapor region and at the liquid-gas surface, but this assumption doesn’t seem to be rougher, than other assumptions on the slip model.

\[
T_1' = T_0' = \frac{\partial T_S}{\partial P} \cdot P' - \frac{G_s}{X_1} \sum_{j=1}^{N_g} \frac{\partial T_S}{\partial c_{l_j}} \left( c_{s_i} - c_{l_j} \right)
\]

When not boiling, no heat exchange between the liquid and the gas is assumed: \( H_1' = 0; \ H_{gl}' = 0 \)

The energy equation for the liquid can be represented in the following form:
\[(W_1 \cdot H_1)' = -W \cdot \bar{G}_s \cdot W_1(T_s, P, c_{s_i})\]
\[(W \cdot X_1 \cdot H_1)' = -W \cdot \bar{G}_s \cdot H_1(T_s, P, c_{s_i})\]
\[X_1 \cdot H_1' = -\bar{G}_s \cdot \left( H_1(T_s, P, c_{s_i}) - H_1(T_s, P, c_{i_j}) \right)\]
\[H_1' \approx -\frac{\bar{G}_s}{X_1} \left( L_s + \left( \frac{\partial H_1}{\partial P} \right)_{T_s, P, c_{s_i}} \right) \cdot (P - P_s)\]
\[C_{P_i} \cdot T_i' + \frac{\partial H_1}{\partial P} \cdot P' = -\frac{\bar{G}_s}{X_1} \left( L_s + \left( \frac{\partial H_1}{\partial P} \right)_{T_s, P, c_{s_i}} \right) \cdot (P - P_s) - \sum_{i=1}^{N_k} \frac{\partial H_1}{\partial c_{i_j}} \cdot (c_{s_i} - c_{i_j})\]

When boiling (i.e. under \( P - P_{g0} \leq P_s \))
\[\bar{G}_s = -X_1 \cdot \frac{C_{P_i} \cdot \frac{\partial T_s}{\partial P} + \frac{\partial H_1}{\partial P}}{L_s} \cdot P', \quad (8)\]

where
\[\bar{L}_s = H_1(T_s, P_s, c_{s_i}) - H_1(T_s, P_s, c_{i_j}) + \left( \frac{\partial H_1}{\partial P} \right)_{T_s, P, c_{s_i}} \cdot (P - P_s) - \sum_{i=1}^{N_k} \frac{\partial H_1}{\partial c_{i_j}} \cdot (c_{s_i} - c_{i_j})\]

Whence we can obtain the following equation for gas temperature of flashing gas-liquid mixture:
\[T_{g'} = \frac{-\frac{\partial H_1}{\partial P} + \bar{C}_{p_g} \cdot \left( T_{g'} - T_s \right) X_1 \cdot \frac{C_{P_i} \cdot \frac{\partial T_s}{\partial P} + \frac{\partial H_1}{\partial P}}{L_s}}{C_{p_g}} \cdot P'\]

where \(\bar{C}_{p_g} = C_{p_g} \left( \frac{T_{g'} + T_s}{2}, c_{g', 1}, P \right)\).

**Equilibrium flash curve calculation**

The temperatures of the liquid and gas region, vapor quantity, phases' compositions and, therefore, all the physical properties of the mixture are functions only of the pressure. So, we can preset a dependency of the most parameters, which are needed for calculating pressure alteration along the tube length (4).
Simulation of Flow in Vent System

if \( P > P_s \rightarrow \frac{dT_{gv}}{dP} = -\frac{\partial H_1}{\partial P} \), else \( \frac{dT_{gv}}{dP} = -\frac{\partial H_1 + \theta_{p_{gv}} \cdot (T_{gv} - T_S)}{C_{p_{gv}}} \cdot \frac{X_1}{X_{gv}} \cdot \frac{C_P \cdot \partial T_S}{\partial P} + \frac{\partial H_1}{\partial P} \)

if \( P > P_s \rightarrow \frac{dT_l}{dP} = -\frac{\partial H_1 + 1}{\partial P} \), else \( \frac{dT_l}{dP} = \frac{\partial T_S}{\partial P} - \frac{\partial X_{gv}}{\partial P} \cdot N_k \sum_{j=1}^{N_k-1} \frac{\partial T_S}{\partial c_i} \cdot (c_j - c_i) \)

if \( P > P_s \rightarrow \frac{dX_{gv}}{dP} = 0, \) else \( \frac{dX_{gv}}{dP} = -X_1 \cdot \frac{C_P \cdot \partial T_S + \partial H_1}{L_s} \)

\[
\frac{dc_{l_i}}{dP} = (c_{l_i} - c_{l_i}) \cdot \frac{dX_{gv}}{dP}
\]

\[
\frac{dc_{g_i}}{dP} = (c_{g_i} - c_{g_i}) \cdot \frac{dX_{gv}}{dP}
\]

for \( i = 1,...,N_k \) \( c_{g_i} \neq 0, \)

for \( i = N_k + 1,...,N_g \) \( c_{g_i} = 0 \)

\( N_g = N_k + N_g \)

where \( L_s = H_1(T_S, P_S, c_{g_i}) - H_1(T_S, P_S, c_{l_i}) \)

\[
L_s = \begin{pmatrix} \frac{\partial H_1}{\partial P} \bigg|_{T_S, P_S, c_{g_i}} & \frac{\partial H_1}{\partial P} \bigg|_{T_S, P_S, c_{l_i}} \end{pmatrix} \cdot \left( P - P_S \right) - \sum_{i=1}^{N_k-1} \frac{\partial H_1}{\partial c_i} \cdot (c_j - c_i)
\]

The results of solving (9) and can be tabulated for further interpolating. In the same way we can calculate and tabulate all properties of gas and liquid and values of \( \dot{v} (5), \rho_h (6, 7) \) as a functions of pressure only.

**Slip correlation**

\[
\phi_f^2 = \left( 1 - X_{gv} \right) \cdot \left( 1 + \Psi \right)^2
\]

\[
\Psi = \frac{s \cdot 1 - X}{v}
\]

\[
r = \frac{V_s}{V_f}
\]

For homogeneous case

\[
\phi_f^2 = \frac{1}{2} \cdot f_1 \cdot \left( \frac{V_1 \cdot X_1 + V_{vg} \cdot X_{vg}}{D_c \cdot A^2} \right)^2
\]

For purely turbulent mode we can use generalized expression [1]:
\[ s = c \left[ \left( 1 - X_{gv} \right)^{a-1} \cdot \left( \frac{v_l}{v_{gv}} \right)^{b+1} \cdot \left( \frac{\mu_l}{\mu_{gv}} \right)^z \right] \]  
\( (10) \)

For the Lockhart—Martinelli correlation, there exist more detailed expressions for different modes of a flow in both phases (see ниже).

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>A</th>
<th>B</th>
<th>Z</th>
</tr>
</thead>
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<tr>
<td>Homogeneous</td>
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<td>1.0</td>
<td>-1.0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Friction losses**

For a laminar flow (Reynolds number less than 2100), friction losses are calculated using the following relations for Fanning friction factor \( f \) (section 3-5-2):

\[ f = \frac{16}{Re}, \]  
\( (11) \)

where

\[ Re = \frac{W \cdot D}{S \cdot \mu} \]  
\( (12) \)

Viscosity of two-phase mixture can be evaluated (especially for homogeneous case) by using the following expression: \( \mu = \alpha \cdot \mu_g + (1 - \alpha) \cdot \mu_l \), where \( \mu_g, \mu_l \) are the dynamic viscosity of gas and liquid phases, respectively. As a rule, we use properties of liquid for the slip model, but for the Lochhart—Martinelli slip ratio the fanning factor must be calculated for both phases separately concerning partial mass flux of liquid or gas, respectively.

For a turbulent flow, friction losses may be computed using one of the following relations for the fanning factor \( f \) (section 3-5-2, [1]):

- Traditional Fanning Factor correlation for commercial rough pipe

\[ f = 0.0035 + \left( \frac{0.264}{Re^{0.42}} \right) \]  
\( (13) \)

- Chen equation

\[ f^{-\frac{1}{2}} = -4.0 \cdot \log_{10} \left[ \frac{\varepsilon}{D} \cdot \frac{1}{3.7065} \cdot \left( \frac{1}{5.0452 \cdot \frac{\psi}{Re}} \right) \right] \]  
\( (14) \)

where \( \psi = \log_{10} \left[ 0.3539 \cdot \left( \frac{\varepsilon}{D} \right)^{1.1098} + 5.8506 \cdot Re^{-0.8981} \right] \)

- Darcy – Weilbach constant fanning factor
- Churchill fanning factor

If \( Re < 1000 \) (laminar flow)
If Re > 10000 (turbulent flow)

\[ f = f_{\text{turb}} = 2 \left( \log \frac{0.27 \frac{v}{D}}{\frac{7}{Re} + \left( \frac{7}{Re} \right)^{0.9}} \right)^{-2} \]

Else

\[ f_{\text{trans}} = \frac{Re^2}{17606361} \]

\[ f = \left[ f_{\text{lam}}^{12} + \left( f_{\text{trans}}^{-8} + f_{\text{turb}}^{-8} \right)^{-1.5} \right]^{0.8333} \]

Note that if the Lockhart – Martinelli slip ratio is selected, the fanning factor is calculated in other way (see the following section).

**Lockhart—Martinelli slip correlation**

The procedure for estimating fanning factor \( f \) for the Lockhart—Martinelli slip ratio substantially differs from the algorithm for other models.

- Define the liquid discharge \( W_l = W \cdot X_l \)
- Calculate \( Re_l = \frac{W_l \cdot D}{A \cdot \mu_l} \)
- Define the gas discharge \( W_g = W \cdot X_g \)
- Calculate \( Re_g = \frac{W_g \cdot D_g}{A \cdot \mu_g} \)

Verify the modes in the phases (the flux is laminar if Re < 1000 and turbulent otherwise\(^7\)). If both phases are in turbulent mode, apply the generalized form (10), else the flux is turbulent with respect to gas and viscous with respect to liquid\(^8\):

1. Depending on \( Re_l \) calculate \( f_l \) by formulas,
2. Calculate the friction losses \( F_{\text{fric}} = \frac{2 \cdot f_l \cdot W_l^2}{D_e \cdot \rho_l \cdot A^2} \)
3. Depending on \( Re_g \) calculate \( f_g \) by formulas (13) or (14)
4. Calculate the friction losses \( F_{\text{fric}} = \frac{2 \cdot f_g \cdot W_g^2}{D_e \cdot \rho_g \cdot A^2} \)
5. Calculate parameter \( \chi^2 = \frac{F_{\text{fric}}}{F_{\text{fric}}} \)
6. Calculate parameter \( \phi_f^2 = \left( 1 + \chi^{0.5} \right)^4 \)

\(^7\) This is a simplified approach, since in practice another phase can influence the transition

\(^8\) We assume, that other modes are out of interest [1]
7. Calculate void fraction:

\[
\alpha = \begin{cases} 
(1 + \chi^{0.8})^{-0.378} & \text{for } \chi < 10, \\
\exp(-1.165 + 0.59 \cdot \ln \chi - 0.1783 \cdot (\ln \chi)^2) & \text{else}
\end{cases}
\]

The formulas from this step should be smoothly conjugated in domain \(10 < \chi < 12\).  

8. Calculate slip ratio:

\[
s = \frac{X_{\text{g}} \cdot (1 - \alpha) \cdot \frac{v_g}{v_1}}{1 - X_{\text{g}} \cdot \alpha}
\]

**Algorithm for calculation of mass flow rate through vent line**

The equation for distribution of pressure along a vent line should be solved as a boundary problem. Mass flow rate is unknown and is an eigenvalue of this boundary problem.

The first boundary conditions are the inlet conditions, where all parameters of the flow are known.

At the second boundary, we have the condition at the tubing outlet \(P = P_{\text{back}}\) for a subsonic flow or presence of a sonic flow at some location in the vent line. For the last case, we should calculate the minimum mass flow rate when the flow is still sonic at some location in the vent line.

The scheme of the algorithm is presented below.

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9. To avoid multiple calculation of the exponents and logarithms, interpolate from the tables.

10. Besides the inlet quality, which may depend on mass flow rate from the tank according to the coupled inlet quality equation (see [1]).

11. This may occur at local contraction or changing of inclination, where the numerator of the fraction in eq. 4 may become zero simultaneously with its denominator.
Simulation of Flow in Vent System

W_{left} = 0; W_{right} = -1

Inlet conditions

Inlet quality correction
Lengthwise integration

Sonic flow

False

Tail tubing exists

True
Lengthwise integration

W_{left} = W
If W_{right} > 0 then
W = W_{left}(1-w) + W_{right} w
else W = W_{left} / w

Outlet boundary conditions
(P_{end} - P_{next}) \text{sign}(dP/dl) < 0

W_{right} > 0 &
(W_{right} - W_{left})/W < \varepsilon

End of calculations

W_{right} = W;
W = W_{left}(1-) + W_{right} w

Fig. 0-1 Flowchart of the algorithm
Calculation of phase velocities
To estimate velocities of liquid and gas phases following formulas can be used:
\[
\begin{align*}
  u_l &= \frac{g \cdot X_1}{\rho_1 \cdot (1 - \alpha)}; \\
  u_g &= s \cdot u_l
\end{align*}
\]

Simulation of local hydraulic resistances
In addition to lengthy hydraulic elements, such as straight tubes or conic tubes, which can be simulated by (4) directly, there are a set of short elements of vent line (elbow, tee, contractions or expansion, orifice, inlet, outlet and other fittings) that have negligible length and additional local hydraulic resistance. For defining the local hydraulic resistance, the software uses the appropriate loss factor for every non-lengthy element. It may be defined by the Hooper formula
\[
K = \frac{K_1}{Re} + K_2 \left(1 + \frac{K_3}{D}ight)
\]
from [6] or by using the discharge coefficient value (see ниже).

Such elements are simulated using the same equation as for tubes (4), but every element is considered as two tubes. The first tube has the length and constant diameter that are both equal to the inlet diameter of the element with no inclination and with an increased friction factor \(f\) (this factor is replaced by a value of \(K/4\)). The second tube may be conic if the inlet diameter of the element is not equal to the outlet one and have a length equal to the inlet diameter too.

A nozzle, an orifice and valves (nozzle-like elements) are considered as combinations of three sub-elements: a contraction from an inlet to an orifice, an expansion from an orifice to an outlet, and a straight tube with a diameter equal to the diameter of the orifice. The length of the first element is equal to the inlet diameter; the length of the second and third elements are equal to the outlet diameter of the nozzle-like element. If the loss coefficient data of the nozzle-like element are specified, these data are referred to the contraction.

Calculation loss factor on the base of values of discharge coefficients
The discharge coefficient is the correction factor defined by the ratio of the dischargeable theoretical mass flux through the valve. It is related to the valve seat cross section and accounts for the non-ideality of the flow through a valve compared to the flow through an ideal nozzle. Certified values for gas and liquid flow are usually supplied from valve manufacturers. They should be determined experimentally. Valve manufacturers generally provide only values for the single-phase low viscous liquid flow, \(K_{d_{\text{liq}}}\), and single-phase choked gas flow, \(K_{d_{\text{g}}}\). We assume that losses corresponding to \(K_{d_{\text{g}}}\) may be treated as losses of a part of the nozzle-like element before the orifice12, and \(K_{d_{\text{liq}}}\) may be referred to losses in all parts of the element. Let us denote by \(K_{\text{inlet}}\) the loss factor before the orifice, and by \(K_{\text{outlet}}\) the loss factor downstream the orifice. Then it can be easily shown that
\[
K_{\text{inlet}} \approx \frac{1 + \sqrt{1 - 2 \cdot K_{d_{\text{g}}}^2 \cdot \beta^4 \cdot N_y} - K_{d_{\text{g}}}^2 \left(1 - \sqrt{1 - 2 \cdot \beta^4 \cdot N_y}\right)}{2 \cdot K_{d_{\text{g}}}^2 \cdot \beta^4 \cdot N_y}
\]
(15)

and \(K_{\text{outlet}} \approx \frac{1}{K_{d_{\text{g}}}^4 \cdot \beta_{\text{out}}^4} \cdot \left(K_{\text{inlet}} + 0.5\right) \cdot \frac{\beta_{\text{out}}^4}{\beta_{\text{out}}^4 - 1}\)

12 We assume that in a gas flow there is the sonic flow transition (choked flow) that is located at the orifice of the valve or other nozzle-like element, hence the losses downstream the location where velocity attains the sonic value cannot influence the upstream flow.
where $\beta = \frac{A_{\text{orifice}}}{A_{\text{inlet}}}$; $N_r = k \cdot \left( \frac{2}{k+1} \right)^{\frac{k+1}{k-1}}$; $\beta_{\text{outlet}} = \frac{A_{\text{outlet}}}{A_{\text{inlet}}}$.

$K_{\text{outlet}}$ is substituted for the last sub-element, which is specially brought for this purpose. $K_{\text{inlet}}$ is substituted for the contraction sub-element of the nozzle-like element.

This procedure should be applied only when $K_{\text{d}_{\text{liq}}} < K_{\text{d}_{\text{g}}}$. If this relation is not satisfied, then, if the user specified a non-zero value of $K_{\text{d}_{\text{liq}}}$, $K_{\text{inlet}} = \frac{1}{\beta^4} \left( \frac{1}{K_{\text{d}_{\text{liq}}}} - \beta_{\text{outlet}}^4 \right) - 0.5$ and $K_{\text{outlet}} = 0$, else, when only $K_{\text{d}_{\text{g}}}$ was specified, we use (19) for calculation of $K_{\text{inlet}}$ and $K_{\text{outlet}} = 0$.

### Simulation of active devices

#### Nonreclosing devices

For this type of devices, when pressure at the nozzle throat\textsuperscript{13} is less than the activation set pressure, the effective flow area is equal to zero, otherwise the device flow area is gradually increasing for 100 ms after the device starts to open up to the full opening area.

#### Conventional valves

The flow area depends on the complex

$$\Omega = \frac{P_{\text{act}} - P_{\text{bonnet}} - \Delta P_{\text{set}}}{\Delta P_{\text{over}}}$$

where $P_{\text{act}}$ - total hydrodynamic pressure\textsuperscript{14} before valve disk;

$P_{\text{bonnet}}$ - pressure in valve bonnet $P_{\text{bonnet}} \approx P_{\text{bd}}$

$P_{\text{bd}}$ - discharge back pressure at valve outlet;

$\Delta P_{\text{set}}$ - gauge activation set pressure ($P_{\text{set}} - P_{\text{atm}}$);

$\Delta P_{\text{over}} = C_{\text{over}} (P_{\text{set}} - P_{\text{atm}})$ - gauge pressure at valve outlet that forces valve to fully open;

$C_{\text{over}} = \Delta P_{\text{over}} / 100$ and $\Delta P_{\text{over}}$ is overpressure in %.

We use the dependence of effective flow area factor on this complex, which is presented in [5]:

\textsuperscript{13} immediately upstream the valve disk

\textsuperscript{14} Pressure plus kinetic head or stagnation pressure
Effective Flow Area Factor

Fig. 2 Effective flow area at partial lift pressure relief valves with standard trim for liquid service.

Using this dependence we add some conservative factor in the case of sonic flow through a conventional valve. In the future, we are going to take into account the correction factor that should be based on experimental data, just as the correction factor $K_b$ for balanced valves.

**Balanced valves**

Back pressure effects can be minimized by the use of a balanced-type valve, such as Bellows Pressure Relief Valves, etc. In this case, the upper surface of the valve disk is partly isolated from the back pressure. However, a high back pressure can reduce the valve capacity but by a smaller value than for a conventional valve.

For taking into account the effect of increased back pressure on the capacity of a balanced relief valve, the user can force the software to apply an appropriate capacity correction factor due to back pressure from API RP 520 [7]. In this document, there are two correction factors: $K_w$ for incompressible liquid, and $K_b$ for gases. We prefer to consider two cases: subsonic flow and sonic flow (chocked), because it is more convenient for two-phase flow modeling. These correction factors should be applied only when the acting pressure is equal to overpressure in absolute units (full overpressure conditions). Therefore, we can’t use $K_w$ and $K_b$ directly, because BST is a simulator and it must be able to calculate mass flow rate at any reasonable value of acting pressure and any reasonable value of back pressure.

As the first step, let us use the assumption that the effective flow area $K_a$ for balanced valves depends on the complex:

\[
\Omega_b = \frac{P_{act} - P_{set}}{C_{over} \left( P_{set} - P_a \right)} - \frac{\varphi}{C_{over}} \cdot \frac{P_{bd} - P_a}{P_{set} - P_a}
\]

We can fit the parameters of function $K_a = K_a (\Omega_b)$ so that if $P_{act} - P_{set} = C_{over} \left( P_{set} - P_a \right)$, then $K_a = K_b$ for sonic flow and $K_a = K_w$ for subsonic flow.

**Back pressure correction factor for subsonic flow**

For subsonic flow, Vent uses the following function for calculating the flow area factor:

\[
K_a = \Omega_b \left( 1.0 + (1.0 - \Omega_b) \cdot (0.05 + 1.1 \cdot \Omega_b) \right)
\]
Fig. 3 Comparison of capacity correction factor due to back pressure ($K_w$) from [7] and $K_a$, calculated by using (18) at full overpressure conditions.

**Back pressure correction factor for sonic flow**

The following formulae are used for calculation of the flow area factor for balanced valves:

$$K_a = \Omega_b \cdot \left( \Omega_b \cdot b + c \right),$$

(19)

where $b = -1.9033 + 0.0656 \cdot w$; $c = 2.8159 - 0.0648 \cdot w$; $w = (\Delta P_{over} - 10)/6$.

$\Omega_b$ is calculated by using (17) where the value of coefficient $\phi$ is calculated as $\phi = 0.14 + 0.025 \cdot w$

Fig. 4. Comparison of capacity correction factor due to back pressure ($K_a$) from [7] and $K_a$, calculated by using (19) at full overpressure conditions for sonic flow.

**Pilot operated valves**

For pilot operated valves, we don’t apply any correction factor due to back pressure. The flow area factor is calculated by using (16), substituting $P_{atm}$ for $P_{bonnet}$.
**Nomenclature**

- $A$ - area;
- $c_{fj}$ or $c_{lj}$ - mass fraction of the j-th component of a liquid mixture;
- $C_p$ - specific heat capacity under constant pressure;
- $c_{Sj}$ - mass fraction of the j-th component of an equilibrium vapor mixture;
- $D_e$ - equivalent hydraulic diameter;
- $d$ - the differentiation operator;
- $f$ - fanning factor;
- $W$ or $W_v$ - mass flow rate through the vent line, kg/s;
- $H$ - enthalpy;
- $l$ - current length of the flow path (originates at the piping inlet);
- $M$ - mass, kg;
- $m_j$ - mole mass of the j-th component of a mixture;
- $N_k$ - number of liquid components;
- $N_g$ - number of gaseous components;
- $P$ - pressure;
- $Re$ - Reynolds number;
- $T$ - temperature, K;
- $u$ - linear velocity, m/sec;
- $X_{g0}$ - mass fraction of the insoluble gas in a gas—liquid flow;
- $Z$ - elevation of the tube outlet comparing to its inlet, m;
- $\alpha$ - void fraction;
- $\beta_p = \left[ \frac{\partial \ln \rho}{\partial P} \right]_{T = \text{Const} \, \, \, \, C_j = \text{Const}}$ - compressibility factor under fixed temperature and composition;
- $\beta_T = \left[ - \frac{\partial \ln \rho}{\partial T} \right]_{P = \text{Const} \, \, \, \, C_j = \text{Const}}$ - thermal expansion factor under fixed pressure and composition;
- $\varepsilon$ - tube roughness (compared to its diameter);
- $\mu$ - dynamic viscosity;
- $\nu_{gf}$ - difference of gas and liquid specific volumes, $m^3/kg$;
- $\rho$ - density (if subscript not present—the mixture density);
- $g$ - mass discharge through the nozzle\(^{15}\);
- $v$ - specific volume of gas-liquid mixture at the nozzle throat;
- $P_0$ - pressure at the inlet

\(^{15}\) The least area of the nozzle
Pressure at the moment of boiling

Pressure at the nozzle throat

gas adiabatic coefficient

Specific volume of initial liquid (at mean values of temperature and pressure)

Liquid heat capacity (at mean values of temperature and pressure)

Mixture temperature at nozzle inlet

Liquid temperature at nozzle throat at boiling:

Pressure head at initial temperature

Gas quality of the mixture at the vent line inlet

Specific volume of initial gas at vent line inlet

Vaporization heat

The specific volume of gas-liquid mixture is calculated by the following formula:

where:

- $\bar{v}_g$ - specific volume of initial gas at vent line inlet
- $\bar{v}_f$ - specific volume of initial liquid
- $\eta_s$ - vaporization heat
- $\gamma$ - liquid compressibility
- $\Delta P_{\text{over}}$ - overpressure in %
- $P_{\text{bd}}$ - discharge back pressure at valve outlet

$R_v$ - gas constant of vapors

$C_v$ - vapor heat capacity at nozzle throat

$C_f$ - liquid heat capacity at nozzle throat

$\beta_p$ - liquid compressibility
Subscripts

\( b \) - gas phase in the bubble zone
\( e \) - equilibrium
\( ext \) - external
\( f \) or \( l \) - liquid
\( g \) - gas phase or gas region
\( g_0 \) - insoluble gas or adiabatic gas zone in a two-phase flow through a long tube
\( \text{mix} \) - mixture of question
\( s \) - equilibrium vapor
\( v \) - vapor
\( \text{atm} \) or \( a \) - related to atmosphere;
\( \text{bonnet} \) - at valve bonnet

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